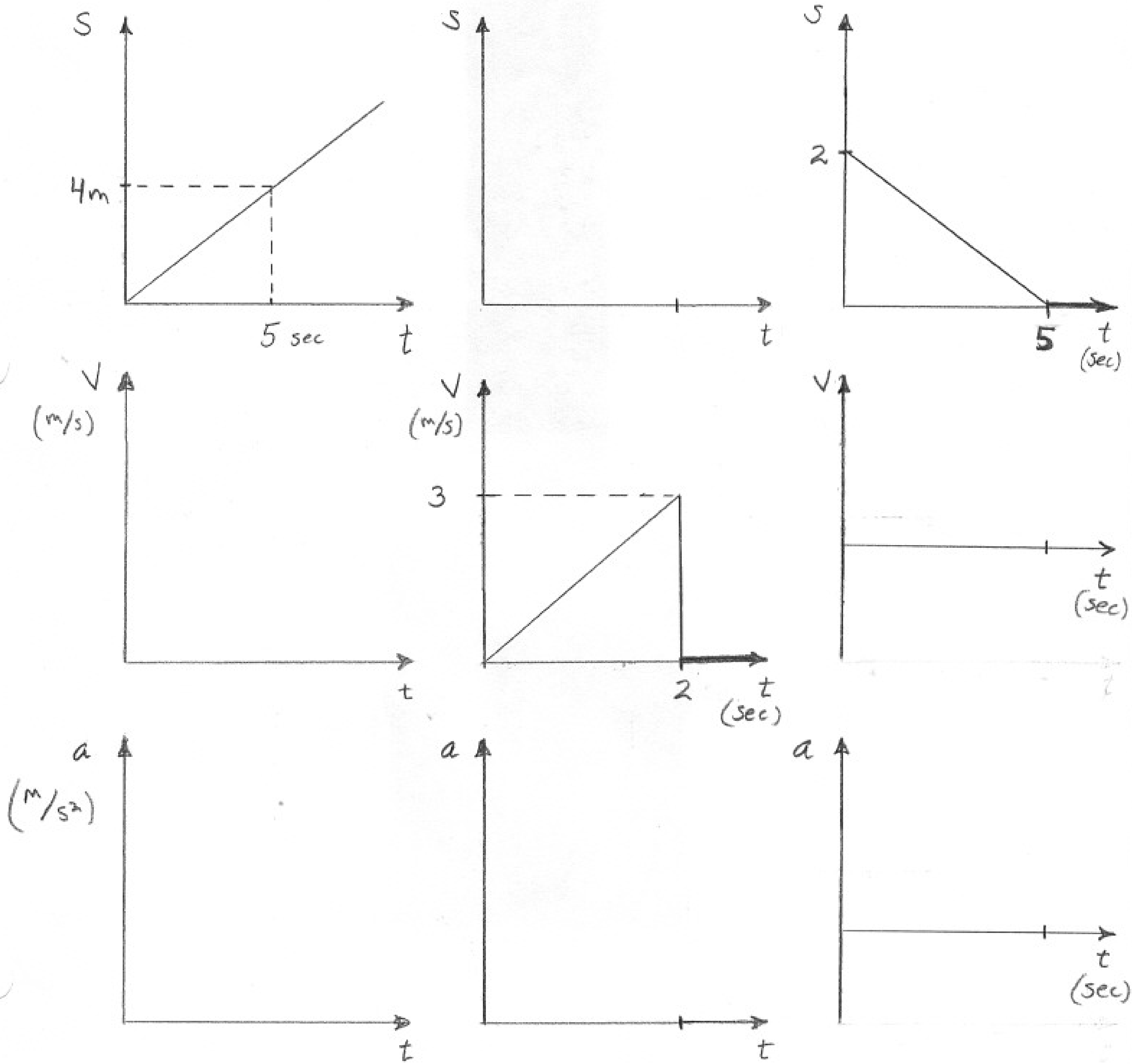
SVAJ Diagrams

- Ultimately we would like to choose a mathematical expression that will allow the follower to exhibit a desired motion. The motion of the cam is analyzed using a SVAJ diagram (S = Displacement, V = Velocity, A = acceleration, J = Jerk). Fill in the blanks below:



- Consider the following cam timing diagram. There are two dwells and we would like to design the cam such that a good motion in the rise and the return.

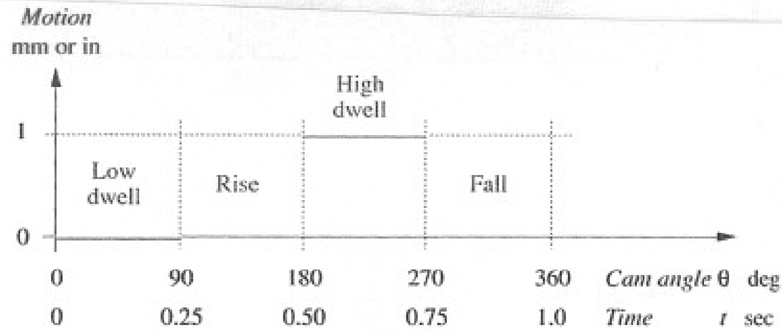


FIGURE 8-7
Cam timing diagram

- Because we're new at designing cams, let's just try a linear function between the low and high dwells.

Displacement (a)

Velocity (b)

Acceleration (c)

Jerk (d)

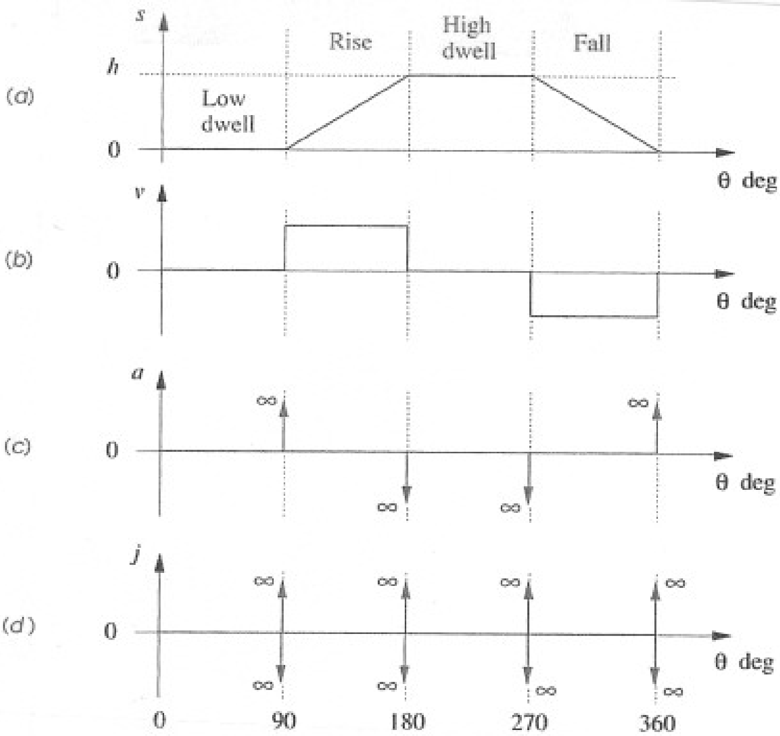


FIGURE 8-8
The *s v a j* diagrams of a "bad" cam design

Infinite acceleration requires an ∞ force since $F = ma$

- Infinite forces are not possible to achieve, but the dynamic forces will be very large at the boundaries and will cause high stresses and rapid wear. Separation between the cam and follower may occur.

- The Fundamental Law of Cam Design: Any cam designed for operation at other than very low speeds must be designed such that
  a) The cam function must be continuous through the first and second derivatives of displacement across the entire 360° interval of motion
  b) The jerk must be finite across the entire 360° interval. The displacement, velocity, and acceleration function must have no discontinuities in them.

- Large or ∞ jerks cause noise and vibration. In order to obey the Fundamental Law of Cam Design, the displacment function needs to be at least a fifth order polynomial. ($V = 4^{th}$ order, $A = 3^{rd}$ order, $J = 2^{nd}$ order and finite)

Simple Harmonic Motion - sinusoids are continuously differentiable

$$s = \frac{h}{2}\left[1 - \cos\left(\pi \frac{\theta}{\beta}\right)\right]$$

$$v = \frac{\pi}{\beta}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$$

$$a = \frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\pi\frac{\theta}{\beta}\right)$$

$$j = -\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$$

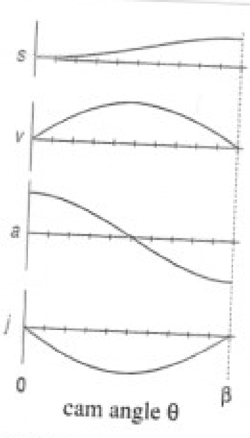


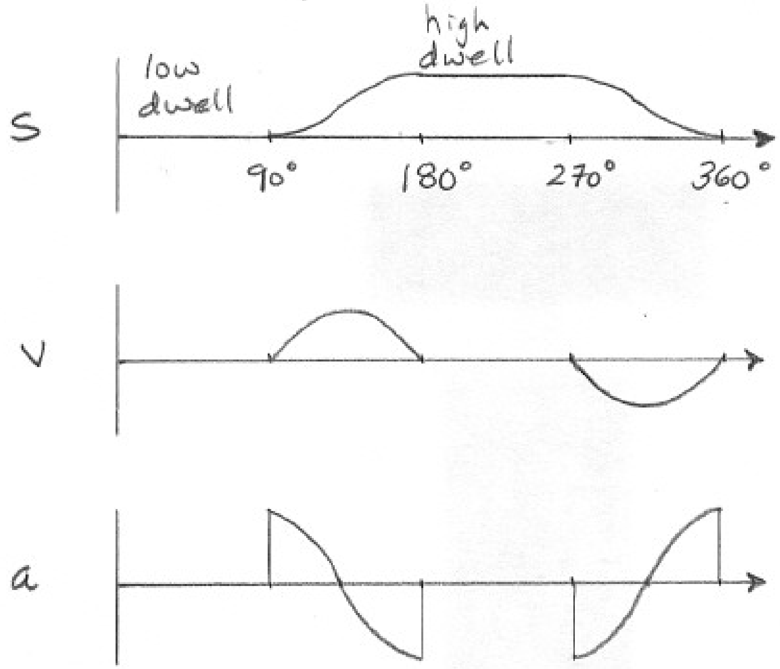
FIGURE 8-9
Simple harmonic motion with dwells has discontinuous acceleration

h = rise

θ = cam shaft angle

β = total ∡ of the rise interval

- If we apply a simple harmonic motion rise and return to our cam timing diagram from p. 3-2



The acceleration is not continuous and so it is a bad cam !!

Cycloidal Displacement (Sinusoidal Acceleration)

If we choose a sinusoid for the acceleration and integrate twice we will obtain the cycloidal displacement function

$$s = h\left[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{\beta}\right)\right]$$

$$v = \frac{h}{\beta}\left[1 - \cos\left(2\pi\frac{\theta}{\beta}\right)\right]$$

$$a = \frac{2\pi h}{\beta^2}\sin\left(2\pi\frac{\theta}{\beta}\right)$$

$$j = 4\pi^2\frac{h}{\beta^3}\cos\left(2\pi\frac{\theta}{\beta}\right)$$

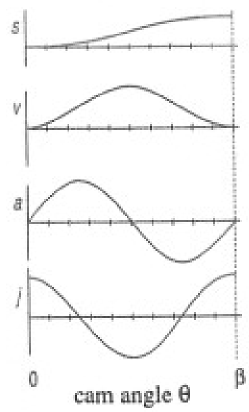
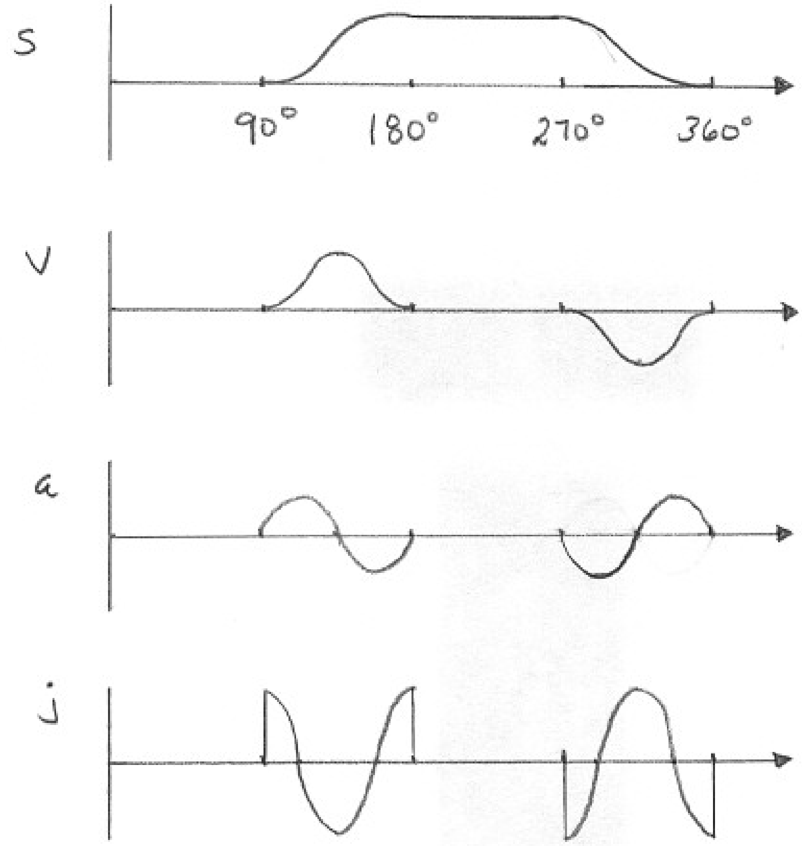


FIGURE 8-12
Sinusoidal acceleration gives cycloidal displacement

Disadvantage: high level of peak acceleration

- If we apply the cycloidal displacement function to our cam timing diagram from p 3-2 →



Cycloidal displacement function

The cam motion is continuous through acceleration and the jerks are finite!

$\therefore$ Acceptable Cam

Modified Trapezoidal Acceleration

This function is designed to minimize extreme accelerations

$0 \le \theta \le \beta/8$ sinusoidal quarter wave rise from zero to $\ddot{y}_{max}$

$\beta/8 \le \theta \le 3\beta/8$ constant $\ddot{y}_{max}$

$3\beta/8 \le \theta \le 5\beta/8$ sinusoidal half wave descent from $\ddot{y}_{max}$ to $\ddot{y}_{min}$

$5\beta/8 \le \theta \le \frac{7\beta}{8}$ constant $\ddot{y}_{min}$

$\frac{7\beta}{8} \le \theta \le \beta$ sinusoidal quarter-wave rise from $\ddot{y}_{min}$ to zero

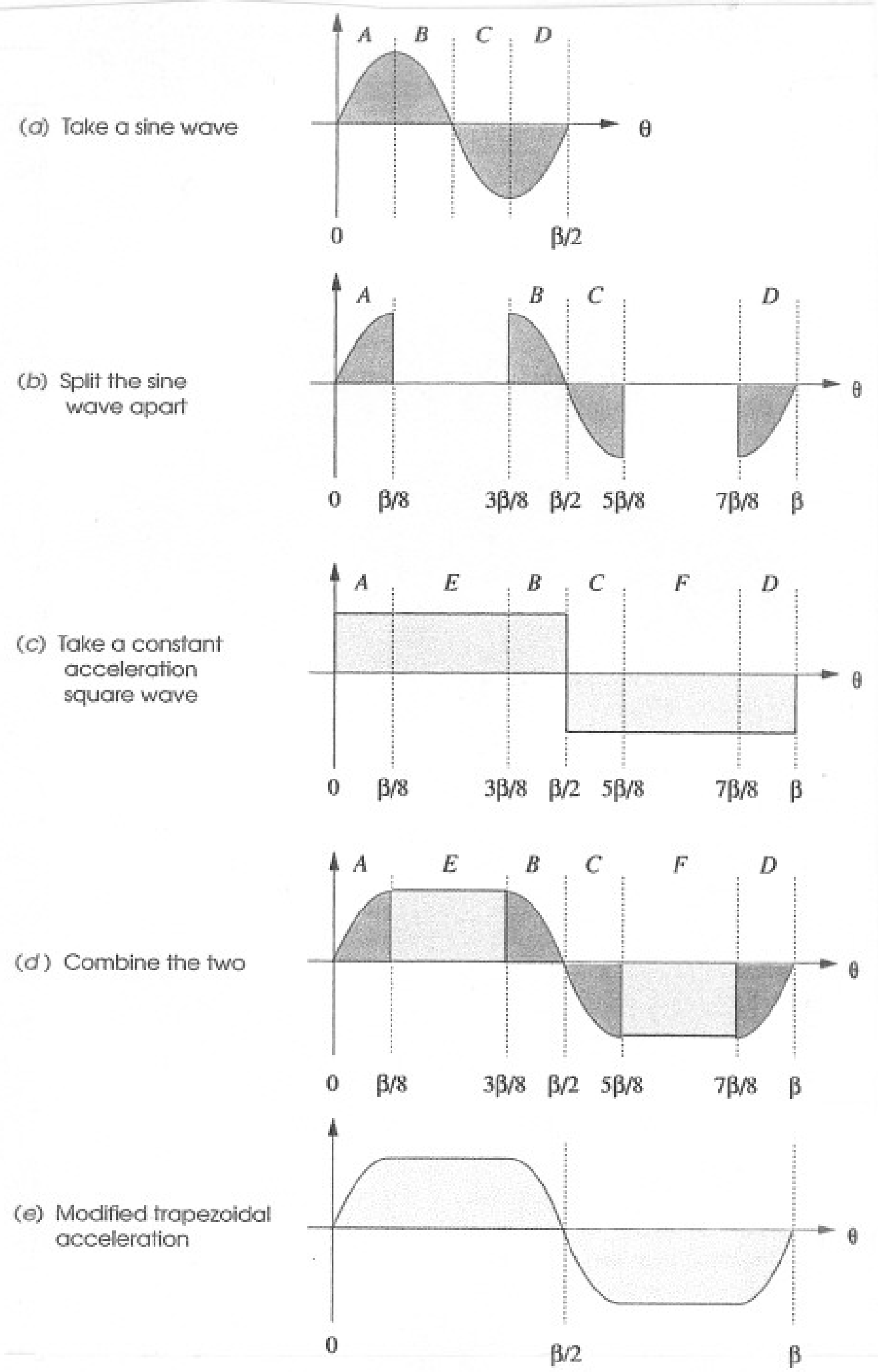


FIGURE 8-15
Creating the modified trapezoidal acceleration function

Modified Sinusoidal Acceleration

Is a combination of cycloidal displacement (smoothness) and modified trapezoidal (minimize peak acceleration). The combination also results in lower peak velocity compared to the other two. The modified sinusoidal is made up of two sinusoids with two different frequencies.

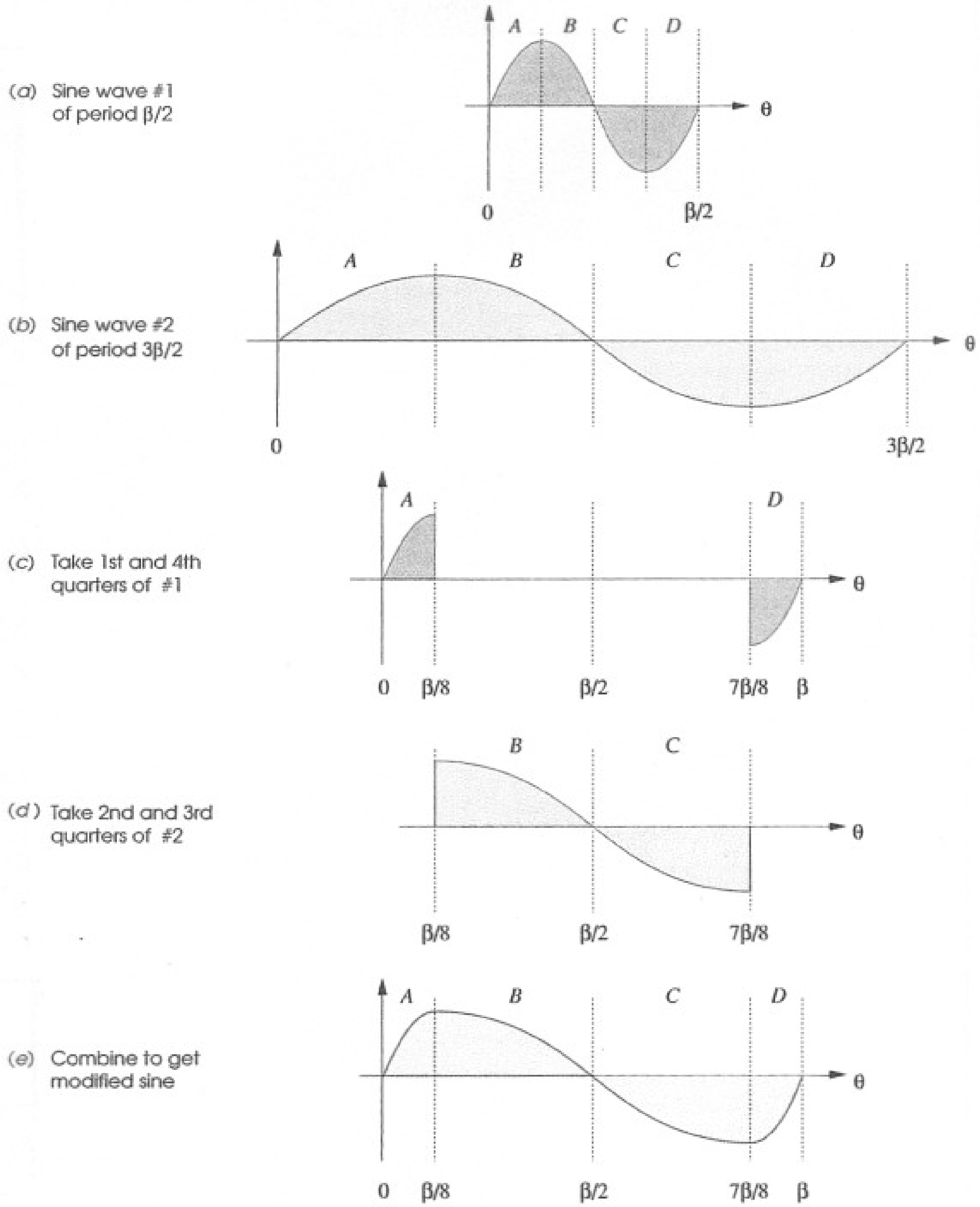


FIGURE 8-16

Creating the modified sine acceleration function